Course Outline

1. Stability & Determinacy of Structures
   1.1 Introduction
   1.2 Stability of Structures,
   1.3 Determinacy of Structures

2. Loads on Structures
   2.1 Dead Load
   2.2 Live Load
   2.3 Environmental Loads (Wind loads, earthquake forces, …)
   2.4 Load Combinations

3. Influence Lines (IL) for Determinate Structures
   3.1 IL for Beams (IL for shear forces and bending moment)
   3.2 IL for paneled girders
   3.3 IL for trusses

4. Deflection of Determinate Structures
   4.1 Direct Integration Method
   4.2 Moment-Area Method
   4.3 Conjugate-Beam Method
   4.4 Method of Virtual Work
   4.5 Graphical Multiplication
   4.6 Castiglione’s Theorem
   4.7 Maxwell Beti law of reciprocal deflection

5. Consistent Deformation Method
   5.1 Indeterminate beams
   5.2 Trusses by Consistent Deformation Method

References:
- Nigussie Tebeje (Prof.), Statically Indeterminate Structural Analysis, 1984;
- Popov, E.P., Mechanics of Materials
- EBCS-1, 1995 (Ethiopian Building Code Standards, part 1- Loadings)
- EBCS-8, 1995 (Ethiopian Building Code Standards, part 8- Design of structures for Earthquake Resistance)
- Lecture note by Abrham Gebre
CHAPTER 1

1. Stability & Determinacy of Structures

1.1 Introduction

A structure refers to a system of connected parts used to support loads. The fundamental purpose of a structure is to transmit loads from the point of application to the point of support and, through the foundations to the ground.

Before going into the analysis of any structure, it is necessary to identify its statical type (classification), i.e., whether it is determinate or indeterminate, stable or unstable. An unstable arrangement of supports and structural members should be avoided.

All structures are subjected to loads from their functions and to other unavoidable loads. Establishment of the loads that act on a structure is one of the most difficult and yet important steps in the design process.

In this chapter; Criteria for statical classification will be established and different structures will be checked for stability and determinacy.

1.2 Stability of Structures

To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or, constrained by their supports. In structural analysis a structure is said to be stable when it can support any possible system of applied loads.

Stability can be divided into two as external and internal.

A structure in which there are insufficient numbers of reactions to prevent motion from taking place is called an unstable structure. This is external instability.

What matters is not only the number of support reactions but also their arrangement. Structures for which the numbers of reaction components are greater than or equal to the number of available equilibrium equations but that are unstable due to arrangement of these reaction components are said to be geometrically unstable.

When the reaction elements are three or more like supports that are either parallel or concurrent, they are not sufficient to maintain static equilibrium.

For the case of parallel reactions, they will offer no resistance to horizontal motion, thus making the arrangement unstable. The point of intersection of the concurrent reactions becomes an instantaneous center of rotation and the system is instantaneously unstable.
The stable fundamental element of a plane truss is a triangular arrangement of three members. A truss may have internal instability if four members are used to form an element.

In conclusion, the stability of structures depends on the number and geometric arrangement of reactions and structural members rather than on the strength of individual member or supports. Despite the possibility that an unstable structure could become stable under a particular system of applied loads, the structure is classified as an unstable structure.

1.3 Determinacy of Structures

When all forces in a structure can be determined strictly from equilibrium equations, the structure is referred to as statically determinate. Structures having more unknown forces than available equilibrium equations are called statically indeterminate.

A statically indeterminate structure is one that cannot be analyzed by the equations of static equilibrium alone. Indeterminacy is introduced in structures on account of functional requirements, limitations on types of framing, need for stiffness and often by the nature of inherent continuity introduced by the type of material used like reinforced concrete.

A structure is statically indeterminate when it possesses more members or is supported by more reactive restraints than are strictly necessary for stability (and equilibrium). The excess members or restraints are called redundant. The degree of indeterminacy is the number of unknowns in excess of the available equilibrium equations. In the analysis of indeterminate structures, therefore, ways of establishing additional equations must be sought. These additional equations may be derived from compatibility of deformation or from conditions of symmetry. This additional task would make the analysis of indeterminate structures more difficult than their determinate counterparts.

Indeterminate structures have some advantages and disadvantages over determinate ones. One obvious disadvantage is the computational difficulty involved when establishing the required additional equations. Another disadvantage is that indeterminate structures will be stressed due to differential settlement of supports, temperature changes and errors in fabrication of members.
On the other hand, however, indeterminate structures are stiffer and in the case of overloads indeterminate structures can provide an advantage of redistribution of loads within the structure.

The indeterminacy of a structure can be external (with respect to reactions) or internal (with respect to member forces). The question of identifying external or internal indeterminacy is largely of academic interest. What is of primary importance is the total degree of indeterminacy. Nevertheless, determining external and internal indeterminacy is desirable as a method to evaluate the total degree of indeterminacy.

A structure is internally indeterminate when it is not possible to determine all internal forces by using the equations of static equilibrium. For the great majority of structures, the question of whether or not they are indeterminate can be decided by inspection. For certain structures this is not so, and for these types rules have to be established. The internal indeterminacy of trusses will be first considered, and then that of continuous frames.

1.4 Criteria for Stability and Determinacy of Structures-Trusses, Beams and Frames

Internal stability of structures and determining which conditions exist in a given case need experience, especially for trusses. In some cases the structure is different from what our mathematical criteria tell us. Therefore, stability of trusses is most easily settled by inspection.

1.4.1 Beams

A beam is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight and external reactions to these loads is called a bending moment. Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e., loads due to an earthquake or wind).

- Stability depends on external supports
- Determinacy relates on the number of available and conditional equations.
  - \( r_a < r; \) structure is statically unstable
  - \( r_a = r; \) structure is statically determinate
  - \( r_a > r; \) structure is statically indeterminate

where \( r_a \) is the available number of reaction components

\( r \) is the minimum number of reaction components required for stability, usually \( 3+n \)

\( n \) is the number of special/conditional equation

Remark: \( r = 3 \) is not a sufficient conditions for stability
1.4.2 Trusses

A simple truss can be made by combining three bars to form a triangle. Stability depends partly on external supports and partly on the arrangement of members or bars. Three reaction components are required for external stability and determinacy of a plane truss without condition equations.

1.4.2.1 External classification

The external statical classification of the structure depends on the total number of reaction components, \( r_a \) and their arrangement. Therefore, the following criteria hold true:

- \( r_a < r \); structure is statically unstable externally
- \( r_a = r \); structure is statically determinate externally
- \( r_a > r \); structure is statically indeterminate externally

where \( r_a \) is the available number of reaction components

\[ r \text{ is the minimum number of reaction components required for external stability, usually } 3+n \]

\[ n \text{ is the number of special/ conditional equation} \]

The condition for \( r_a \geq r \) is necessary but not sufficient conditions for statical classification because the arrangement of the reaction components may render the truss unstable.

1.4.2.2 Internal classification

For internal classification, in addition to the above definition for \( r \); let \( m \) be the total number of bars and \( j \) the total number of joints. Then

\[ 2j = m + r \]

The above equation can be rewritten as:

\[ m = 2j - r \]

In this form, \( m \) is the number of members required to form an internally statically determinate truss that connects \( j \) joints and has \( r \) reaction components required for external stability. If \( m_a \) is the actual number of bar forces in the truss, then the following criteria hold true for internal classification

- \( m_a < m \); truss is statically unstable internally
- \( m_a > m \); truss is statically determinate internally
- \( m_a > m \); truss is statically indeterminate internally

Consider the trusses shown below. The truss shown in fig (a) is stable whereas the truss shown in fig (b) is unstable since the geometric arrangement of the members is not maintained.

Fig (a)       Fig (b)
# Examples

<table>
<thead>
<tr>
<th>Structure Characteristics</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Classification</td>
<td>Classification</td>
</tr>
<tr>
<td>$r_a$</td>
<td>$r$</td>
</tr>
<tr>
<td>$r_a = x$, deter. stable</td>
<td>$r_a = x$, deter. stable</td>
</tr>
<tr>
<td>$r_a &gt; x$, indet. stable</td>
<td>$r_a &gt; x$, indet. stable</td>
</tr>
<tr>
<td>$r_a = x$, deter. stable</td>
<td>$r_a = x$, deter. stable</td>
</tr>
<tr>
<td>$r_a &gt; x$, indet. stable</td>
<td>$r_a &gt; x$, indet. stable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structure</th>
<th>$i$</th>
<th>$m_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>(b)</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>(c)</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>(d)</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>(e)</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>(f)</td>
<td>14</td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structure</th>
<th>$M_y = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g)</td>
<td>$\Sigma P = 0$, $M_y = 0$</td>
</tr>
</tbody>
</table>

Diagram examples showing various structures and their classifications based on the given parameters.
1.4.3 Frames

Frames are composed of continuous members and rigidly connected joints. The degree of indeterminacy (DI) is determined as the difference of the total number of unknown reaction components and the number of static equilibrium equations available. Note that the frame with the hinge has a fourth condition equation, since the bending moment at the hinge must be zero. Stability depends partly on external supports and partly on moment resisting joints.

1.4.3.1 External classification

The external statical classification of the structure depends on the total number of reaction components, \( r_a \) and their arrangement. Therefore, the following criteria hold true:

- \( r_a < r \): structure is statically unstable externally
- \( r_a = r \): structure is statically determinate externally
- \( r_a > r \): structure is statically indeterminate externally

where \( r_a \) is the available number of reaction components

\( r \) is the minimum number of reaction components required for external stability, usually \( 3+n \)

\( n \) is the number of special/conditional equation

\( r_a \geq r \) is necessary but not sufficient conditions for statical classification because the arrangement of the reaction components may render the frame unstable.

1.4.3.2 Internal classification

\((3 + r) < (3j + n)\);

Let \( m_a \)=the actual number of members

\( r \)= the minimum number of independent reaction components required for external stability

\( j \)= the total number joints

\( n \)= number of special/condition equations

Therefore, \( 3m_a + r \)=the number of unknowns

\( 3j+n \)=the number of available equations

Then the following criteria hold true for internal classification of frames

- \((3m_a + r) < (3j + n)\): structure is statically unstable
- \((3m_a + r) = (3j + n)\): structure is statically determinate
- \((3m_a + r) > (3j + n)\): structure is statically indeterminate

Overall classification

The criterion already established for both trusses and frames hold also for investigation of overall effect. To determine the overall classification of a frame, in the above expressions replace \( r \) by \( r_a \).

Note. The number of conditional equation introduced by a hinge joint is equal to the number of members at the joint minus one.
### Examples

<table>
<thead>
<tr>
<th>Structure Characteristics</th>
<th>Overall Classification</th>
<th>External Classification</th>
<th>Classification</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall Classification</td>
<td>(3m + n)</td>
<td>Classification</td>
<td>Classification</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(j + n)</td>
<td>Overall Classification</td>
<td>External Classification</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Classification</td>
<td>Classification</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Determinate, stable</td>
<td>Determinate, stable</td>
</tr>
</tbody>
</table>
CHAPTER 2

2. Loads on Structures

Determination of the loads that act on a structure, evaluation of critical force effects in the member and dimensioning are the most difficult and yet important steps in the overall process of design.

The loads that enter a system are of three different types. Concentrated loads (example a single vehicular wheel load) are those that are applied over a relatively small area. Line loads are distributed along a narrow strip of the structure. The weight of a member itself and the weight of a wall or partition are examples of this type of load. Surface loads are loads that are distributed over an area. The loads on a warehouse floor and the snow load on a roof are examples of surface loads.

The loads that act on a structure can be grouped into three categories: dead loads, live loads, and environmental loads. These categories can be further divided according to the specific nature of the loading. Because the method of analysis is the same for each category of loading, all loads could be combined before the analysis is performed. However, separate analyses for the individual loading cases are usually carried out to facilitate the consideration of various load combinations.

Furthermore loads can be classified based on:

- **Direction**: Gravity/vertical and Lateral/horizontal loads
- **Variation with time**: dead load (permanent) and Live Loads (temporary)
- **Structural Response**: Static (loads applied gradually) and dynamic (loads applied over a short period of time and vary in magnitude with time)

2.1 Dead loads

Dead loads are those that act on the structure as a result of the weight of the structure itself and of the components of the system that are permanent fixtures. As a result, dead loads are characterized as having fixed magnitudes and positions. Examples of dead loads are the weights of the structural members themselves, such as beams and columns, the weights of roof surfaces, floor slabs, ceilings, or permanent partitions, and so on.

The dead loads associated with the structure can be determined if the materials and sizes of the various components are known. Some of standard material unit weights are shown in Table 2.1. (Refer EBCS-1, 1995 Ethiopian Building Code Standards, part 1- Loadings)
### Table 2.1

<table>
<thead>
<tr>
<th>Material</th>
<th>Unit wt. (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Brick</td>
<td>22</td>
</tr>
<tr>
<td>Hollow Concrete Block (HCB)</td>
<td>14-20</td>
</tr>
<tr>
<td>Trachyte (Masonry)</td>
<td>26</td>
</tr>
<tr>
<td>Concrete (with reinforcement)</td>
<td>25</td>
</tr>
<tr>
<td>Steel</td>
<td>77</td>
</tr>
<tr>
<td>Zigba (podacargus Gracillior)</td>
<td>6</td>
</tr>
<tr>
<td>PVC floor covering</td>
<td>16</td>
</tr>
</tbody>
</table>

### 2.2 Live loads

In a general sense, live loads are considered to include all loads on the structure that are not classified as dead loads. However, it has become common to narrow the definition of live loads to include only loads that are produced through the construction, use, or occupancy of the structure and not to include environmental.

These loads are dynamic in character in that they vary both in magnitude and position. Live loads where the dynamic nature has significance because of the rapidity with which change in position occurs are called moving loads, whereas live loads in which change occurs over an extended period of time, or where there is the potential for change whether exercised or not, are referred to as movable loads. Moving loads include vehicular loads on bridges or crane loads in industrial buildings. Another type of live load is a variable load or a time dependent load—that is, one whose magnitude changes with time, such as a load induced through the operation of machinery.

#### 2.2.1 Occupancy live loads

Occupancy live loads for buildings are usually specified in terms of the minimum values that must be used for design purposes. Some representative values are given in Table 2.2.

(Refer EBCS-1, 1995 Ethiopian Building Code Standards, part 1- Loadings)

<table>
<thead>
<tr>
<th>Category</th>
<th>Uniform Load (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private dwelling</td>
<td>1.5</td>
</tr>
<tr>
<td>Bed rooms, class rooms, . . .</td>
<td>2.0</td>
</tr>
<tr>
<td>Offices, Café, . . .</td>
<td>3.0</td>
</tr>
<tr>
<td>Assembly halls, Cinema, . . .</td>
<td>4.0</td>
</tr>
</tbody>
</table>

#### 2.2.2 Traffic Loads for Bridges

Bridges must be designed to support the vehicular loads associated with their functional use and minimum loads are mandated for designed purposes. In the case of highway bridges, these loads are specified in Bridge Design Manuals. The approach is to specify the weights and spacing of axles and wheels for a design truck, a design tandem, and the design lane load. These loadings provide for a set of concentrated loads (which represent a truck type loading) and a uniform load (which simulates a line of vehicles).
2.2.3 Impact loads
Loads that are applied over a very short period of time have a greater effect on the structure than would occur if the same loads were applied statically. The manner in which a load varies with time and the time over which the full load is placed on the structure will determine the factor by which the static response should be increased to obtain the dynamic response.

For building occupancy loads, the minimum design loads normally include adequate allowance for ordinary impact conditions. However, provisions must be made in the structural design for uses and loads that involve unusual vibrations and impact forces. One situation in which an impact effect (IM is defined as the dynamic load allowance) is applied for moving vehicular loads on a highway bridge.

2.3 Environmental loads
Structures experience numerous loading conditions as a result of the environment in which they exist. These are Snow and Ice Loads, Roof loads, Wind loads and Earthquake Loads.

2.3.1 Snow and Ice Loads
The procedure for establishing the static snow loads on a building is normally based on ground snow loads and an appropriate ground-to-roof conversion.

The distribution of snow on a roof is complex, and many different approaches are used. Factors considered in calculating snow and ice loads are location, exposure factor, thermal factor, the effects of unloaded portions of roof, unbalanced or nonuniform loads on various roof configurations, drifting, sliding snow, and extra loads induced by rain on snow.

Snow loads are not normally considered in bridge design because they are usually small when compared with other loadings on the structure. However, ice loads can be appreciable on bridge structures. The icing not only creates loads on the structure but also increases the member sizes, which, in turn, increases the magnitude of the wind induced loads.

2.3.2 Rain loads
Roof loads that result from the accumulation of rainwater on flat roofs can be a serious problem. This condition is produced by the ponding that occurs when the water accumulates faster than it runs off, either because of the intensity of the rainfall or because of the inadequacy or blockage of the drainage system. The real danger is that as ponding occurs the roof deflects into a dished configuration, which can accommodate more water, and thus greater loads result.

The best way to prevent the problem is to provide a modest slope to the roof (0.25 in. per ft or 2cm. per m or more) and to design an adequate drainage system. In addition to the primary drainage, there should be a secondary system to preclude the accumulation of standing water above a certain level.
2.3.3 Wind loads

The wind loads that act on a structure result from movement of the air against the obstructing surfaces. Wind effects induce forces, vibrations, and in some cases instabilities in the overall structure as well as its non-structural components. These wind effects depend on the wind speed, mass density of the air, location and geometry of the structure, and vibrational characteristics of the system.

The design wind pressure that is used to establish the wind load on a structure is directly related to velocity pressure (q) and is given by:

\[ q = \frac{1}{2} \rho v^2 \]

Where \( p \) is the mass density of air, and \( v \) is the wind velocity.

**Wind Forces According to EBSC-1, 1995**

*Wind Pressure*: The external and internal wind pressures are given as:

\[ W_e = q_{ref} c_e(z_e)c_{pe} \]

\[ W_i = q_{ref} c_e(z_e)c_{pi} \]

Where \( q_{ref} \) is the reference wind pressure; \( W_e \) and \( W_i \) are the external and internal pressures; \( c_e(z_e) \) and \( c_e(z_i) \) are the external and internal exposure coefficients; \( c_{pe} \) and \( c_{pi} \) are the external and internal pressure coefficients.

*Reference Wind Pressure*: The reference wind pressure is given by:

\[ q_{ref} = \frac{1}{2} \rho v_{ref}^2 \]

Where \( p \) is air density and \( v_{ref} \) is the reference wind velocity.

The air density is a function of altitude and depends on the temperature and pressure to be expected in the region during storms. A temperature of 20°C has been selected as appropriate for Ethiopia and the variation of mean atmospheric pressure with altitude is given in Table 2.3.

<table>
<thead>
<tr>
<th>Site Altitude (m) Above Sea Level</th>
<th>0</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Air density, ( \rho )</strong></td>
<td>1.20</td>
<td>1.12</td>
<td>1.06</td>
<td>1.00</td>
<td>0.94</td>
</tr>
</tbody>
</table>

*Reference Wind Velocity*: The reference wind velocity is taken as 22m/s.

*Exposure Coefficient*: takes into the account the effects of terrain, topography, and elevation.

\[ C_e(z) = f^n(\text{terrain category, topographic coefficient, elevation}) \]
**Terrain Category:** The terrain category attempts to take into account the effect of the land coverage, and is given below. The terrain type is classified into 4 groups as follows:

- **Category I:** Lakes with at least 5 km fetch upwind and smooth flat country without obstacles.
- **Category II:** Farmland with boundary hedges, occasional small farm structure, houses or trees.
- **Category III:** Suburban or industrial areas and permanent forests.
- **Category IV:** Urban areas in which at least 15% of the surface is covered with buildings and their average height.

**Topography Coefficient:** The topography coefficient $C_t$ accounts for the increase in mean wind speed over isolated hills and escarpments and mountainous regions. It is defined by:

- $C_t = 1$ for $\Phi < 0.05$
- $C_t = 1 + 2S\Phi$ for $0.05 \leq \Phi < 0.3$
- $C_t = 1 + 0.6S$ for $\Phi > 0.3$

Where: $S$ is a factor to be obtained by interpolation from the value of $s=1.0$ at the crest of a hill or escarpment and the value of $S=0$ at the boundary on the topography affected zone, $\Phi$ is the upwind slope in the wind direction.

![Diagram of terrain types](image1)

**Factor $S$: Cliffs and escarpments**

![Diagram of terrain types](image2)

**Factor $S$: Hills and ridges**
**Pressure Coefficient**: The shape factor takes into account the effect of shape of structure on the pressure distribution.

The external pressure coefficients cpe for buildings and individual parts of building depend on the size of the loaded area A. They are given for loaded area A of 1m² and 10m² in the relevant tables for the appropriate building configuration as cpe.1 and cpe.10, respectively. For areas between 1m² and 10m², values are obtained by linear interpolation. That is:

\[
\begin{align*}
C_{pe} &= c_{pe.1} & \text{for } A \leq 1\text{m}^2 \\
C_{pe} &= c_{pe.1} + \left(c_{pe.10} - c_{pe.1}\right) \log_{10} A & \text{for } 1\text{m}^2 < A < 10\text{m}^2 \\
C_{pe} &= c_{pe.10} & \text{for } A \geq 10\text{m}^2
\end{align*}
\]

The values of pressure coefficient are applicable to buildings.

Values of external pressure coefficients for different cases are given in Table A.1 to Table A.5 of EBCS-1, 1995.

### 2.3.4 Earthquake Loads

A common dynamic loading that structures must resist is that associated with earthquake motions. Here, loads are not applied to the structure in the normal fashion. Instead, the base of the structure is subjected to a sudden movement. Since the upper portion of the structure resists motion because of its inertia, a deformation is induced in the structure. This deformation, in turn, induces a horizontal vibration that causes horizontal shear forces throughout the structure.

It results from the acceleration of the supporting earth. Movement of the ground during EQ in the direction parallel to the ground surface has the most damaging effect on structures. The resulting earthquake loads are dependent on the nature of the ground movement and the inertia response characteristics of the structure. The computation of lateral loads due to EQ and load distribution to various levels of a building frame as of EBCS-8, 1995 (Ethiopian Building Code Standards part 8- Design of structures for Earthquake Resistance) is presented below.

\[
F_b = S_d(T_1)W
\]

\[
S_d(T_1) = \alpha \beta \gamma
\]

Where:

- **F_b** = Total lateral load on the structure (seismic base shear)
- \(S_d(T_1)\) = Ordinate of the design spectrum
- \(T_1\) = Fundamental period
- \(W\) = Seismic dead load \(DL\) (+25% for storage and warehouses)
- \(\alpha = \alpha_0 I\), \(\alpha\) is the ratio of the design bedrock acceleration to the acceleration of gravity, \(g\)
- \(I = \) importance of a structure
- \(\beta = 1.25 \sqrt{T/2.5} \leq 2.5\), \(\beta\) is the design response factor
- \(\gamma\) = accounts the ductility level (behavior factor)
\[ \alpha_0 = \ \text{Bed rock acceleration for the site} \]
\[ S = \ \text{Site coefficient.} \]
\[ T_i = C_i H^{3/4}, \quad C_i = 0.075 \quad \text{H = height of the building above base in m} \]
\[ C_i = \begin{cases} 
0.085, & \text{for steel moment resisting frames} \\
0.075, & \text{or for RC moment resisting frames, and eccentrically braced frames} \\
0.050, & \text{for other buildings} 
\end{cases} \]

Thus, the total lateral load distributed to various level of the building frame is given by the formula:

\[ F_i = \left( F_b - F_{t} \right) \frac{w_i h_i}{\sum w_i h_i} \]

\[ h_i = \text{height of each floor level} \]

\[ F_t = 0.07 T_i F_b \]

\[ w_i = \text{Weight of the structure at each floor level} \]

Therefore, \( F_i \) at level \( i \) distributed in accordance with mass distribution on that level.

The parameters \( \alpha_0 \), \( I \), \( S \) and \( \gamma \) are obtained as follows.

i) Bed rock acceleration \( (\alpha_0) \)

Areas are subdivided into seismic zones depending on the local hazard.

<table>
<thead>
<tr>
<th>Zone</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0.1</td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Examples of seismic zones of some towns of Ethiopia are shown in the following table.

<table>
<thead>
<tr>
<th>Zone</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Towns</td>
<td>Awassa, Mekele, Nazreth, Asaita</td>
<td>Assela, Dila</td>
<td>Addis Ababa, Dire-Dawa</td>
<td>Ambo, Axum, Jima, Jijiga</td>
<td>Assossa, Bahir-dar, Gondor</td>
</tr>
</tbody>
</table>
ii) Importance of a structure (I)

\[ I = \begin{align*}
1.4 & \rightarrow \text{hospitals, fire stations, power plants} \\
1.2 & \rightarrow \text{schools, halls, …} \\
1.0 & \rightarrow \text{ordinary buildings} \\
0.8 & \rightarrow \text{buildings of minor importance}
\end{align*} \]

iii) Site coefficient (S)

For different soil types, the value of S is given:
- Subsoil \( A \) – 1.0 \( \rightarrow \) rock, stiff deposits of sand, gravel or over consolidated clay
- \( B \) – 1.2 \( \rightarrow \) deep deposits of medium dense sand, gravel or medium stiff clays, …
- \( C \) – 1.5 \( \rightarrow \) loose cohesion less soil deposits with or without soft cohesive layers,

iv) The behavior factor (\( \gamma \))

It depends on the structural system:
- for concrete structures \( \gamma \leq 0.70 \)
- for steel structures, it ranges from 0.17 to 1.0
- for timber structures it varies from 0.30 to 1.0
2.4 Load Combinations

Ultimate Design Load

The ultimate design load acting on a member will be the summation of the relevant characteristic load combinations multiplied by their respective partial safety factors. Thus, the ultimate design load for the combination of dead and imposed loads will be expressed as follows:

\[
\text{Ultimate design load} = \gamma_f \times \text{characteristic load}
\]

Partial Safety Factors for Load

In practice the applied load may be greater than the characteristic load for any of the following reasons:

a. Calculation errors
b. Constructional inaccuracies
c. Unforeseen increases in load.

To allow for these the respective characteristic loads are multiplied by a partial safety factor \(\gamma_f\) to give the ultimate design load appropriate to the limit state being considered. That is,

\[
\text{Ultimate design load} = \gamma_f \times \text{characteristic load}
\]

Load combinations depend on the design philosophy adopted.

Load Combinations for Ultimate Limit States (ULS)

- Permanent action \((G_k)\) and only one variable action \((Q_{ki})\)
  \[F_d = 1.3G_k + 1.6Q_{ki}\]
- Permanent action \((G_k)\) and two or more variable actions
  \[F_d = 1.3G_k + 1.35\Sigma Q_{ki}\]
- Permanent action, variable action and accidental (seismic) action
  \[F_d = G_k + Q_{ki} + A_{Ed} = 0.75 (1.3G_k + 1.6 Q_k) + A_{Ed}\]

Load Combinations for Serviceability Limit States (SLS)

- Permanent action \((G_k)\) and only one variable action \((Q_{k1})\)
  \[F_d = G_k + Q_{k1}\]
- Permanent action \((G_k)\) and two or more variable actions
  \[F_d = G_k + 0.9\Sigma Q_{ki}\]

The final design of a structure must be consistent with the most critical combination of loads that the structure is to support. However, some judgment is necessary in selecting loading conditions that can reasonably be combined. Obviously, the maximum effects of all loading conditions should not be combined because it is unlikely that they will all occur simultaneously.
CHAPTER 3

3. Influence Lines for Determinate Structures

Introduction
An influence line for a given function, such as a reaction, axial force, shear force, or bending moment, is a graph that shows the variation of that function at any given point on a structure due to the application of a unit load at any point on the structure.

An influence line for a function differs from a shear, axial or bending moment diagram. Influence lines can be generated by independently applying a unit load at several points on a structure and determining the value of the function due to this load, i.e. shear, axial, and moment at the desired location. The calculated values for each function are then plotted where the load was applied and then connected together to generate the influence line for the function.

Influence Lines for a Simple Beam
For illustration consider the simply supported beam shown in figure 3.1 and draw the influence lines for the reactions $R_A$, $R_C$, and the shear and bending moment at point $B$, of the simply supported beam shown.

![Figure 3.1](image)

i. Influence Line for Reaction at $A$, $R_A$

The influence line for a reaction at a support is found by independently applying a unit load at several points on the structure and determining, through statics, what the resulting reaction at the support will be for each case.

![Figure 3.1a](image)

$\sum M_C = 0$

$R_A(L)=l(L-x) \rightarrow R_A = (L-x)/L$

If the unit load is applied at $A$, the reaction at $A$ will be equal to unity. Similarly, if the unit load is applied at $B$ (at $x=L_1$), the reaction at $A$ will be equal to $(L-L_1)/L = L'/L$, and if the unit load is applied at $C$ (at $x=L$), the reaction at $A$ will be equal to 0.
ii. **Influence Line for Reaction at C, RC**

From figure 3.1a above,
\[ \sum M_A = 0 \]
\[ R_C(L) = 1(x) \rightarrow R_C = X/L \]
At \( X=0 \), \( R_C=0 \) and at \( X=L \), \( R_C=1 \)

iii. **Influence Line for Shear at B**

The influence line for the shear at point B can be found by developing equations for the shear at the section using statics. This can be accomplished as follows:

a) if the load moves from A to B, consider figure 3.1d
\[ \sum F_y = 0 \rightarrow V_B = R_A - 1 \]
But, \( R_A = (L-x)/L \)
Therefore the shear force at B, \( V_B \) becomes
\[ V_B = -(L-x)/L = -X/L \]
At \( X=0 \), \( V_B=0 \) and at \( X=L_1 \), \( V_B = -L_1/L \)

b) if the load moves from B to C, figure 3.1e
\[ \sum F_y = 0 \rightarrow V_B = 1 - R_C \]
But, \( R_C = X/L \)
Up on substitution, the shear force at B, \( V_B \) becomes:
\[ V_B = (L-X)/L \]
At \( X=L_1 \), \( V_B = (L-L_1)/L = L_r/L \) and at \( X=L \), \( V_B = 0 \)
iv. Influence line for Moment at B

The influence line for the moment at point B can be found by using statics to develop equations for the moment at the point of interest, due to a unit load acting at any location on the structure.

a) if the load moves from A to B, consider figure 3.1d

\[ \sum M_B = 0 \rightarrow M_B + 1(L - X) - R_A(L_i) = 0 \]

\[ BM_B = R_i L_i = \frac{X}{L} L_r \]

at \( x = 0 \) \( BM_B = 0 \)

at \( x = L_i \) \( BM_B = \frac{L_i L_r}{L} \)

\[ \overline{m} = \frac{L_i L_r}{L} \]

\[ \begin{align*}
\text{Fig 3.1g Influence line for Bending moment at point B.}
\end{align*} \]

b) if the load moves from B to C, figure 3.1e

\[ \sum M_B = 0 \rightarrow R_C(L_r) - 1(X - L_i) - M_B = 0 \]

\[ BM_B = R_i L_i = \frac{L - x}{L} L_i \]

at \( x = L_i \) \( BM_B = \frac{L_r L_i}{L} \)

at \( x = L \) \( BM_B = 0 \)

- Determination of a stress at a paint due to different loads using influence curve

i) Series of concentrated loads say \( P_1 \) and \( P_2 \) as shown.

The product of the load \( P_1 \) and the ordinate of the influence line at the position of the respective load shall give the magnitude of the stress at the section due to the induced load.

In the particular case

\[ SFa = S_1 P_1 + S_2 P_2 \]

Where \( S_1 \) and \( S_2 \) represent the ordinates of the IL with appropriate sign for \( SFa \), at the respective positions of the loads \( P_1 \) and \( P_2 \).
ii) Uniformly distributed load \( w \), say for a length \( d \), position as shown

\[
\frac{dx}{dx} \quad w \text{ KN/m}
\]

Consider a length \( xd \) and force acting \( F = wdx \).

\[ SF_a \text{ due to total load over length } d = \sum_{i=1}^{d} wd_i, \quad Si = \sum_{i=1}^{d} Si dx \]

Where: \( \sum Si dx \) is the area under \( w \) of the IL

Hence, the product of \( W \) and the area under the influence curve give the stress under consideration due to a uniformly distributed load:

- **Determination of position of moving load system for maximum value of a particular function:**
  
  To establish criteria for position of LL (living load) the different type of load system are considered.
  
  A. Single moving load.
  
  - Place the load at the point of the ordinate of the IL for that function is a maximum
  
  B. Uniform load longer than the span of the structure for which the ordinates to IL for that function have the same sign.
  
  - Place the load over all those portion of the structure for which the ordinates to IL for that function have the same sign.
  
  C. Uniform load less than the span:
  
  - In this particular case, lets consider the shape of influence curve
  
  - IL for SF type of simple beam:
    
    - Place the head of the load at the section and let the load covers the left portion for maximum negative and the right portion for maximum value.
  
  - IL for BM type of simple beam
    
    - Critical position may be obtained if the load is placed in such a way that the section divides the load in the same ratio as the section divides the span.
D. Series of concentrated leads at fixed distance a part:-

Here also we need to consider the shape of IL

- **Shear force type**

  \[
  \frac{P_n}{\ell_r} \geq \frac{\sum P}{\ell}
  \]

  The maximum \(SF\) occurs the first load of the system which give an intensity of leading equal to or greater than the average intensity of loading for the loads an the span moving from left for negative \(SF\) and from right for positive \(SF\)

- **BM at a section of simple beam:-**

  \[
  w_1 - \text{resultant of the force at the right}
  \]

  for the load to the left of the section \(\frac{w_1}{\ell_1} > \frac{w_2}{\ell_2}\), while for the load to the right of the section \(\frac{w_1}{\ell_1} < \frac{w_2}{\ell_2}\),
E. Absolute maximum stress for simple beam:
   i) Absolute maximum $SF$
      - Occurs at a section immediately adjacent to one of the sections.
   ii) Absolute max $BM$
      a) For simple or uniform load. Absolute maximum $BM$ occurs at mid span
      b) for simple or concentrated load.

   \[
   \frac{3}{2} IL \text{ for paneled girders}
   \]

   In bridge construction, the live loads are usually transmitted to the main beams or girders through floor beams. Consider the following figure to construct IL for shear force at point $P$.

   The absolute maximum $BM$ occurs under any particular load when the center of the span is midway between that load and the resultant ($R$) of all the loads on the span.

   \[ x = \frac{l - d}{2} \]

   For $u=1$ between joint 1 and 2,

   \[ R_1 = \frac{l_2 - x}{l_2 - l_1} \quad \text{and} \quad R_2 = \frac{x - l_1}{l_2 - l_1} \]

   Consider the lower beam

   \[ R_A \quad P \quad R_1 \quad R_2 \quad R_B \]
From statics,
\[ R_A = \frac{l - x}{L} \quad \text{and} \quad R_B = \frac{x}{L} \]

\[ SF_P = R_A - R_1 \]

\[ SF_A = \left( \frac{l - x}{L} \right) - \left( \frac{l_2 - x}{l_2 - l_1} \right) \]

At \( x = l_1 \), \( SF_A = -\frac{l_1}{L} \) at \( x = l_2 = l_1 + l_3 \), \( SF_A = L - \frac{l_2}{L} \)

3.3 IL for trusses
As the loads are gradually applied to a truss at panel points (joints), the same procedure as that used for constructing ILs for paneled girders is applicable.

The basic assumption to construct ILs for trusses and girders is the stringers act as simple beams between the adjacent floor beams so that the IL will be a straight line between any two adjacent panel points/joints.
CHAPTER 4

4. Deflection of Determinate Structures

Beams
As a flexural structure responds to loading, it assumes an equilibrium configuration under the combined action of the loads and reactions. Corresponding to this external state of equilibrium, there is a distribution of internal shears and bending moments throughout the structure. These internal actions would normally be shown in the form of shear and moment diagrams for each member. At any given point within the structure, there is a curvature that is consistent with the moment at that point. These curvatures accumulate as angle changes along the member lengths, causing each member to deflect into a flexed or bent configuration. The individual members of the deformed structure must fit together in a compatible fashion, and all the displacement boundary conditions must be satisfied.

The design of beams is not complete until the amount of deflection has been determined for the specified loads. Failure to control beam deflection within proper limits in building results in cracks in walls and ceilings. If these deflections are excessive, they may result in psychological frustration of occupants and stacking of opening beams in machines. The deflection of a beam depends on:

- The strength of the material
- The dimension of the beam
- The magnitude of the loads
- Types of supports

Deflection of structures can be calculated in the following methods:

- Direct Integration Method
- Moment-Area Method
- Conjugate-Beam Method
- Method of Virtual Work
- Graphical Multiplication
- Castiglione’s Theorem
- Maxwell Beti law of reciprocal deflection

4.2 Direct Integration Method

Differential equation of the elastic curve
For a flexural member, the force-deformation relationships must relate member-end moments to the corresponding member-end rotations. In a more general formulation, member-end shears and transverse deflections would be included, but they are omitted in our present discussion.

Consider a member under flexural action as shown in Fig. 4.1a, and from it isolate the beam element ab as shown in Fig. 4.1b, which is subjected to the positive moment M. As the element bends, the top fibers are compressed while the bottom fibers are elongated. In between, there is a
longitudinal fiber whose length remains unchanged: this fiber is the so-called neutral fiber of the member.

It is assumed that as the beam deflects, plane cross sections before bending remain plane after bending. For the element ab, extensions of lines through cross sections at a and b intersect at point 0, the center of curvature, forming the angle de. If tangents to the deflected neutral fiber are constructed at points a and b, it is evident that de also measures the angular deformation over the length of the beam element. The line eb is constructed parallel to the deflected cross section at a creating triangle bde. Then, for small angles, comparing triangles bde and oab, we have

\[
d\theta = \frac{dx}{\rho} = \frac{dl}{c}
\]  

(1)

where \(\rho\) is the radius of curvature of the element, c is the distance from the neutral fiber to the topmost fiber, and dl is the shortening of that top fiber. Equation (1) can be rewritten in the form

\[
\frac{dl}{dx} = \frac{c}{\rho} = \varepsilon
\]  

(2)

From stress strain relationship, \(\sigma = E\varepsilon\)

Thus equation 2 becomes

\[
\sigma = -E \frac{c}{\rho}
\]  

(3)

Figure 4.1 Flexural deformations of beam element. (a) Deflected beam. (b) Beam element subjected to moment.
Where $\sigma$ is the stress in the top fiber and $E$ is Young's modulus. The minus sign is introduced to indicate that the element is being compressed (negative stress). This fiber stress could also be expressed by the familiar expression from basic mechanics that

$$\sigma = \frac{Mc}{I}$$  \hspace{1cm} (4)

where $M$ is the moment acting on the element and $I$ is the moment of inertia. Here the negative sign indicates that a positive moment produces a negative (compressive) stress on the top fiber.

Equating equation 3 and 4, yields

$$\frac{1}{\rho} = \frac{M}{EI}$$

but from elementary calculus, the curvature of a plane curve is given by:

$$k = \frac{1}{\rho} = \frac{d^2y}{dx^2} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2}$$

where $k$ is defined as the curvature.

Practically the elastic curves of beams are very flat (slope of the deflected structure) thus their slope $dy/dx$ is negligible as compared to unity. Thus, $k = \frac{1}{\rho} = \frac{d^2y}{dx^2}$

Therefore, $\frac{d^2y}{dx^2} = \frac{M}{EI}$ (This is the basic differential equation of the elastic curve).

From the relationships were developed between static functions load, shear, and bending moment, these static functions can be written as follows:

$$V(x) = \frac{dM}{dx} = EI \frac{d^3y}{dx^3} \quad \text{and} \quad q(x) = \frac{dV}{dx} = EI \frac{d^4y}{dx^4}$$

The family of relationships is extended to include the deformation quantities of slope and deflection.

$$d\theta = \frac{M}{EI} dx$$

The systematic solution of beam deflection problems conditions is called boundary conditions where the constants of integration are determined from the boundary and continuity conditions on $V$, $M$, $\theta$, and $y$.

Figure 4.2 below serves to summarize the family of relationships spanning from the load intensity $p$ through the displacement $y$. In this figure, a simply supported beam is subjected to a general loading and the member responses are shown through plotted functions for load, shear, moment, slope, and deflection.
4.2 Moment-Area Method

The integration method is of greatest value when the loading is such as to produce a moment diagram that is a continuous function over the entire length of the beam.

When concentrated loads occur along the span, or internal reaction points exist, then the moment diagram has discontinuities. This leads to additional constants of integration that are evaluated by applying continuity conditions at the points of moment discontinuity.

If deflection of certain selected points only is to be determined, the moment area method is preferable and also more efficient for beams with several discontinuities due to change in loading and variation in the rigidity of the beams.

Consider the beam structure of Fig. 4.3a, which is shown in a deflected configuration under the action of the applied loads. An enlarged view of a portion of the deflected structure between points A and B is isolated in Fig. 4.3b.
Within region AB, an element of length \( dx \) with tangents with the deflected member constructed at each end of the element. The angle between these end tangents, which represents the angle change that occurs over the length \( dx \), is denoted \( d\theta \). This angle change is given by

\[
d\theta = \frac{M}{EI} dx
\]

Where \( M \) and \( I \) are the bending moment and moment of inertia at point \( x \), respectively, and \( E \) is the modulus of elasticity of the material. If the \( M/EI \) values are plotted as shown in Fig. 4.3c, it is clear that \( d\theta \) is given by the shaded area.

The total angle change that occurs between tangents constructed at points A and B is labeled \( \theta_B^A \) in Fig. 4.3b. This angle is the slope at B relative to the slope at A; it results from the summation of the incremental angle changes between A and B and is given by

\[
\theta_B^A = \int_A^B d\theta
\]

Upon substitution it becomes

\[
\theta_B^A = \int_A^B \frac{M}{EI} dx
\]

This equation is the basis for the first moment-area theorem, which can be stated as follows:
The angle change between points A and B on the deflected structure, or the slope at point B relative to the slope at point A, is given by the area under the M/EI diagram between these two points.

Examination of Fig. 4.3b shows that if the tangents to the element of length dx are extended, they embrace an intercept of \( d\Delta \) on a vertical line through point B. For small angles, this intercept is given by

\[
d\Delta = xd\theta
\]

Up on Substitution, we obtain \( d\Delta = \frac{x}{E} \frac{M}{EI} \) which shows that the intercept \( d\Delta \) is given by the static moment of the shaded area of the M/EI diagram taken about an axis through point B. The accumulation of these intercepts for all increments between points A and B gives

\[
\Delta_B^4 = \int_A^B d\Delta
\]

where \( \Delta_B^4 \) is the vertical displacement of point B on the deflected structure with respect to a line drawn tangent to the structure at point A. Upon substitution the above equation yields:

\[
\Delta_B^4 = \int_A^B \frac{M}{EI} \frac{M}{dx}
\]

This equation is the basis for the second moment-area theorem, which can be stated as follows:

The deflection of point B on the deflected structure with respect to a line drawn tangent to point A on the structure is given by the static moment of the area under the M/EI diagram between points A and B taken about an axis through point B.

It is emphasized that the deflection quantities that are determined by using the second moment-area theorem are normal to the original orientation of the member.

### 4.3 Conjugate-Beam Method

Another method which is derived from the moment area principle is the Conjugate-Beam Method. The problem of beam statics is governed by the following equation.

\[
q(x) = \frac{dV}{dx} = \frac{d^2M}{dx^2}
\]

This is a second-order linear differential equation, and the solution is the familiar shear and moment diagram problem: starting with the load, the first integration gives the shear and the second integration gives the moment. Similarly, the beam deflection problem is governed by

\[
\frac{M}{EI} = \frac{d\theta}{dx} = \frac{d^2y}{dx^2}
\]

This is also a second-order linear differential equation. Here, we start with the curvature, M/EI; the
first integration yields the slope, and the second integration gives the deflection. This observation would enable us utilize the semi graphical method to obtain y’(x) and y(x).

To use the conjugate beam method, an imaginary beam (conjugate beam) is conceived that has the same length as the real beam and has a set of boundary conditions and internal continuity conditions on shear and moment that match the corresponding real beam boundary conditions and internal continuity conditions on slope and deflection.

The following table shows that the geometric conditions of the real beam and the corresponding force conditions of the conjugate beam.

Unlike the moment-area method, an orderly sign convention can be employed with the conjugate beam method. If positive curvature (M/EI) is applied as positive (upward) load intensity on the conjugate beam will correspond to the correct signs of the resulting shears and moments on the conjugate beam correspond to the correct signs of the slope and deflection, respectively, on the real beam.
4.4 The Method of Virtual Work

If a deformable structure, in equilibrium and sustaining a given system loads is subjected to a virtual deformation as result of some additional action the external virtual work of the given system of loads is equal to the internal virtual work of the stress caused by the given system of load

i.e External virtual work = Internal virtual work

i) Deflection resulting from axial strains

Consider the truss shown. Suppose Δ direction

- Superpose the real loads next. Thus, the fictitious force will move through a distance Δ.

Therefore, the external virtual work is Δ × 1.

On the other hand, let u be the fictitious bar force resulting from the action of unit force

The strain in each member due to the superposed load is $SL/E_A$.

The internal virtual work due to u an each member $= u \frac{SL}{EA}$

Then the total internal virtual work for entire truss $= \sum u \frac{SL}{EA}$.

Hence, by the principle of virtual work, $1 \times \Delta = \sum u \frac{SL}{EA}$

Note:- If the sign of $\sum u \frac{SL}{EA}$ is positive, then the actual deflection has the same sense as the unit force otherwise opposite to it. However, it is important that the proper sign for tension (+) and compression (-) be consistent throughout due to the computation for u and S.

The fictitious member force u is a deflection coefficient. So when multiplied by the change in length of the corresponding member, this coefficient u will give the effect of the change in length of that member on to the required deflection component.
ii) **Deflection resulting from flexural strains**

1×Δ and 1×θ expression for external virtual work.

![Beam Diagram]

For the expression of internal virtual work consider the simple beam shown.

Suppose ΔA is required
- A unit fictitious load is applied at A as shown
- Let Mₓ be internal moment at x due to the fictitious unit load.

Due to the flexural strains resulting from the application of real loads, the internal fictitious moment on one face of the DE is caused to rotate through some virtual angle Bₓ relative to the other face

Internal virtual work for a length \( dx = mₓBₓ \)

iii) **Deflection resulting from torsional strains**

Consider the cantilever shaft under real torque \( T \).

![Shaft Diagram]

Due to \( T \), the free end rotates a \( \phi \) rad. The fictitious torque \( t \) will move through \( \phi \) radians

\[
t⋅\phi = t \frac{tdₓ}{JG},
\]

for a length \( dx \), for entire length \( L \),

Internal virtual work

\[
\int_0^L t \frac{Tdₓ}{JG}
\]

Thus, \( 1×Δ = \int_0^L t \frac{Tdₓ}{JG} \) and \( 1×θ = \int_0^L t \frac{Tdₓ}{JG} \)
4.5 Graphical Multiplication

Displacement computations may be simplified considerably by the introduction of a special technique known as the graphical multiplication method for the calculation of product integrals belonging to the type \( \int_{0}^{L} m M dx \)

Note that the integral contains the products of two ordinates to the m and M curves. For the technique to apply, at least one of the curves must be a straight line while the other may be bounded by any curve.

Since \( m = x \tan \alpha \)

\[
\int_{0}^{L} m M dx = \tan \alpha \int_{0}^{L} x M dx = \tan \alpha \int_{0}^{L} x dA_M
\]

Where \( M dx = dA_M \) represents the differential of the area \( A_M \) bounded by the M - curve.

Consequently, \( \int_{0}^{L} x dA_M = A_M x_c \)

where \( Q = \) statical moment of the area \( A_M \) about \( 0-0' \) axis

\( X_c = \) abscissa of the centroid of \( A_M \)

Therefore, \( \int_{0}^{L} m M dx = x_c (\tan \alpha) A_M = A_M x_c \)

Hence, the product of the multiplication of two graphs, one of which at least is bounded by a straight line, equals the area bounded by the area of the graph of arbitrary outline multiplied by the ordinate to the first graph measured along the vertical passing through the centroid of
the second one. It should be noted that the ordinate $mc$ must be measured on the graph bounded by a straight line.

When a graph of a trapezoid shape or of a quadratic parabola has to be multiplied by another trapezoid graph, it is convenient to subdivide into triangular and parabolic segments.

For rapid computations, evaluation of the areas and positions of centroids of different shapes must be readily available. In the case of beams of variable section, $EI$ must be included under the sign. A numerical integration could be applied to the integral

$$\int_{0}^{l} m\left(\frac{M}{EI}\right) dx$$

Alternatively, the value of the integral can be found from

$$\int_{0}^{l} m\left(\frac{M}{EI}\right) dx = A_M^* m_c^*$$

where $A_M^*$ is the area under the $M/EI$ curve and $m_c^*$ is the ordinate of the $m$-curve corresponding to the centroid of the area of the $M/EI$ curve.

The areas and positions of their centroids for simple curves are given in the following Table.

<table>
<thead>
<tr>
<th>Shape of graph</th>
<th>Area</th>
<th>Position of centroid</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Trapezoid" /></td>
<td>$hL$</td>
<td>$L/2$ $L/2$</td>
</tr>
<tr>
<td><img src="image2" alt="Triangular" /></td>
<td>$hL/2$</td>
<td>$L/3$ $2L/3$</td>
</tr>
<tr>
<td><img src="image3" alt="Parabola" /></td>
<td>$hL/3$</td>
<td>$L/4$ $3L/4$</td>
</tr>
<tr>
<td><img src="image4" alt="Cubic Parabola" /></td>
<td>$hL/4$</td>
<td>$L/5$ $4L/5$</td>
</tr>
<tr>
<td><img src="image5" alt="Quadratic Parabola" /></td>
<td>$2hL/3$</td>
<td>$3L/8$ $5L/8$</td>
</tr>
</tbody>
</table>
4.6 Castiglione’s Theorem

The deflection component of the pt of application of an action on a structure in the direction of that action will be obtained by evaluating the 1st partial derivative of the total internal strain energy of the structure wrt the applied action.

\[ i.e \Delta = \frac{2w}{2p} \]

Consider the simple beam loaded as shown below

Internal strain energy = external work for gradual application (a)

\[ \Rightarrow w = \frac{p\Delta}{2} + \frac{O\gamma}{2} \quad - - - (1) \]

When \( dp \) is added \( lP \Rightarrow \) the resulting increment of internal energy

\[ dw = \left( p + \frac{dp}{2} \right) d\Delta + Qd\gamma \quad \text{neglecting differential of higher order} \]

\[ dw = pd\Delta + Qd\gamma \quad - - - (2) \]

If \( p \) and \( dp \) and \( Q \) were gradually and simultaneously placed,

Internal strain energy

\[ w' = \frac{p\Delta}{2} + \frac{dp\Delta}{2} + \frac{pd\Delta}{2} + \frac{O\gamma}{2} + \frac{Qd\gamma}{2} \quad - - - (3) \]

But \( dw = w' - w \)

\[ dw = \frac{dp\Delta}{2} + \frac{pd\Delta}{2} + \frac{Qd\gamma}{2} \quad - - - (4) \quad \{ \text{sub 1 from 3} \} \]

Upon simplifying using 2)

\[ \Delta = \frac{dw}{dp}, \text{ thus the thm hold} \]
Since more than one action will usually be applied to the structure, the general expression for deflection by this theorem should be written as a partial derivative.

\[ \Delta = \frac{2w}{2p} \]

Moreover, this may be simplified as, say for internal work resulting from bending as.

\[ \Delta = \frac{2}{2p} \int \frac{M^2}{2EI} \, dx = \int M \frac{2M}{2p} \frac{dx}{EI} \]

Likewise, for axial strain

\[ \Delta = \sum S \frac{2s}{2p} \frac{L}{AE} \]

For rotational deflection, the partial derivatives will be taken wrt a moment and is written as

\[ \theta = \int M \frac{2M}{2Mo} \frac{dx}{EI} \]

Similar expression for deflection resulting from torsional strains could be applied as

\[ \Delta = \int T \frac{2T}{2p} \frac{dx}{JG} \quad \text{and} \quad \theta = \int T \frac{2T}{2Mo} \frac{dx}{JG} \]

During computation, if the sign of the answer is negative, the actual deflection is opposite to the sense of the action wrt which the derivative taken. If a deflection component is required for a pt where no action is applied, or if an action exist at the pt but not in the direction of the desired deflection component, then an imaginary action is applied at the pt and in the desired direction until the derivative of the total internal strain energy has been found. The imaginary action is then reduced to zero.

4.7 Maxwell Beti law of reciprocal deflection

The principle of reciprocal deflections is one of the most important in the theory of structures and has a wide application in the analysis of statically indeterminate structures. Maxwell's Reciprocal Theorem may be stated as follows:

In any elastic system the displacement caused by a unit load along the line of action of another unit load is equal to the displacement due to the second unit load along the line of action of the first load. The theorem for the beam shown below can be expressed

\[ \delta_{ab} = \int m_a \left( \frac{m_a dx}{EI} \right) = \int m_b \left( \frac{m_a dx}{EI} \right) = \delta_{ba} \]
In a similar manner for the beam in Fig. 2.54b, Maxwell's Theorem states that the slope (rotational displacement) at point b due to a unit force at a is equal to the linear displacement at 0, due to a unit couple at b.

\[
\delta_{ab} = \int m_y \left( \frac{m_y \, dx}{EI} \right) = \int m_a \left( \frac{m_y \, dx}{EI} \right) = \delta_{ba}
\]

That is, \( \delta_{ab} = \int m_y \left( \frac{m_y \, dx}{EI} \right) = \int m_a \left( \frac{m_y \, dx}{EI} \right) = \delta_{ba} \)

It will be found that Maxwell's Reciprocal Theorem is perfectly general. Using the symbol \( \delta \) to indicate any type of displacement, the theorem can be written as

\[
\delta_{ij} = \delta_{ji}
\]

This equation expresses Maxwell's law. That is, for a linearly elastic structure, the displacement at point i due to a unit load at point j is equal to the displacement at point j due to a unit load at point i.
CHAPTER 5

5. The Consistent Deformation Method

The method of consistent deformations, or sometimes referred to as the force or flexibility method, is one of the several techniques available to analyze indeterminate structures. The following is the procedure that describes the concept of this method for analyzing externally indeterminate structures with single or double degrees of indeterminacy.

**Principle:** - Given a set of forces on a structure, the reactions must assume such a value as are not only in static equilibrium with the applied forces but also satisfy the conditions of geometry at the supports as well as the indeterminate points of the structure.

This method involves with the replacement of redundant supports or restraints by unknown actions in such a way that one obtain a basic determinate structure under the action of the applied loading and these unknown reactions or redundant. Then, the derived basic determinate structure must still satisfy the physical requirements at the location of the excess supports now replace by redundant reactions.

5.1 Beams by Consistent Deformation

The basic procedures to solve intermediate beams by the method of consistent deformation method are as follows:

- determine the degree of indeterminacy
- select redundant and remove restraint
- determine reactions and draw moment diagram for the primary structure
- calculate deformation at redundant
- write consistent deformation equation
- solve consistent deformation equation
- determine support reactions
- draw moment, shear, and axial load diagrams

For illustration consider the beam loaded as shown
Basic determinate beam under applied loading is shown below.

\[ \Delta_B = \frac{PL}{2EI} \cdot \frac{L}{2} \cdot \frac{1}{6}L \Rightarrow \Delta_B = \frac{5}{48} \frac{PL^3}{EI} \]

\[ \delta_B = \frac{R_B L^3}{3EI} \]

\[ \Rightarrow R_B = \frac{5}{16}P \]

From statics, \( R_B = \frac{11}{16}P \) and \( M_A = \frac{3}{16}PL \)

### 5.2 Trusses by Consistent Deformation

The method essentially consists of choosing a basic determinate truss (structure) on which the applied loading and the redundant force act and the applying conditions of geometry requiring the deflection in the direction of the redundant force must be zero or specified value. Once the redundant are determined, the member forces and other desired reaction components can be determined by the principle of superposition.
The given truss is indeterminate to the 1st degree internally and to the 1st degree externally. A basic determinate structure shown is selected with external redundant $H_D$ (the horizontal reaction) and the internal redundant (diagonal member $f$).

From the geometry of the original structure (fig 5.2):

\[
\Delta_{10} = \sum u_1 \frac{SL}{AE}, \quad \Delta_{20} = \sum u_2 \frac{SL}{AE}
\]

\[
\delta_{11} = \sum u_1 \frac{SL}{AE}, \quad \delta_{22} = \sum u_2 \frac{u_2L}{AE}
\]

\[
\delta_{12} = \delta_{21} = \sum u_1 \frac{u_2L}{AE}
\]

For this the following format is of great value

<table>
<thead>
<tr>
<th>Member</th>
<th>Length</th>
<th>Area</th>
<th>$S$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$E\Delta_{10}$</th>
<th>$E\Delta_{20}$</th>
<th>$E\delta_{11}$</th>
<th>$E\delta_{21}$</th>
<th>$E\delta_{22}$</th>
<th>$F_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\frac{u_1SL}{A}$</td>
<td>$\frac{u_2SL}{A}$</td>
<td>$\frac{u_1u_1L}{A}$</td>
<td>$\frac{u_1u_2L}{A}$</td>
<td>$\frac{u_2u_2L}{A}$</td>
<td></td>
</tr>
<tr>
<td>ab</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The final member force is obtained as $F_i = F_{oi} + H_D u_{1i} + f u_{2i}$.